

# All You Need Is Cards

Brain Epstein

What follows is a peek into the lives of four lads during one of their most creative periods. Several effects with cards are considered, peppered with fantastic and fabulous references. All will be explained in due course.

## You Know My Suit (Look Up the Number)

The boys were still groggy after a late night of recording when they re-assembled at noon for rehearsals. John was the last to arrive, greeting his three friends with as enthusiastic a “Good morning, good morning,” as he could muster. George was squatting on the floor, reading *Quartet*, a collection of short stories by the paperback writer Somerset Maugham, which Paul had given him. Nobody was in a hurry to get to work, and there were no objections when George pulled out a pack of cards.

“John,” he began, throwing clubs aside. “Guess who has real magical powers? It’s not just his lucky rings! Yes, it’s our little Richard!”

“A-wop bop a-loo bop a-lop bam boom! I thought that was me,” Paul said, clearly a bit miffed.

“I’m so tired,” said John, trying to look interested. “I haven’t slept a wink. But hey, hey, hey, hey, I don’t want to spoil the party. So you’re on a roll, Ringo? Okay, give us your best trick then, for Pete’s sake.” John, Paul and George were now seated in a circle on the floor.

George handed John ten clubs in order, from Ace to Ten, as Ringo and Paul ambled off for a quick cup of tea. “Ringo!” George called after them, “Don’t be long; I need you.” He turned back to John. “Mix these up as much as you want while Ringo is gone; then give them back to me.”

“Hey bulldog!” said John, as he looked through the cards, “We’ve certainly played our share of clubs over the years!” When he had thoroughly jumbled the cards he handed them back to George, who looked through them briefly, and dealt them out face up, in the order in which John had handed them to him. He then silently turned four of them face down.

“Mr. H performs his tricks without a sound,” observed John. “Mr. H will demonstrate,” George chimed in with a grin, “Four Somersets he’ll undertake on solid ground.” He picked up his book again and studied it intently until Paul and Ringo returned.

The cards were lined up in this order: a face-down card, followed by the 10, 5, 6, 8, another face-down, the 2, two face-down cards, and the 4. “You know what to do,” said George. Ringo surveyed the cards silently, his lips moving noticeably as he tried to concentrate.

“I know what they are!” said John excitedly. The others looked up. “Number 9, Number 9, Number 9, Number 9...” he said slowly, in a monotonous voice. Everybody groaned. Then Ringo finally spoke: “Boys, the hidden cards are the Ace, 9, 7 and 3, in that order.”

“What did I tell you? I was right about one of them!” shouted John, leaping up with a laugh. He turned over the four cards, gasping when he saw that Ringo had got them all right. “By jingo, Ringo, it seems that you have the touch, all right,” he said, obviously quite impressed.

“With a little luck...” said Paul under his breath. “Wait—can you do it again?” John interjected. “Not a second time. Happiness is a warm pack. I bet he can’t do to it any time at all.”

Paul mixed up the first ten hearts from the pack while Ringo went off to get more sugar for his tea. “Sugar daddy’s lovely hearts/clubs banned,” mocked Paul, as he handed the cards to George, who glanced through them, as before, and dealt them out without changing the order. However, this time he turned five of them face down with a sly grin. When Ringo returned he saw a face-down card followed by the 7, three face-down cards, the 3, Ace, another face-down card, and then the 5 and 6. “That’s five you expect me to get right,” he said indignantly. George smiled sweetly. “Considering how much sugar you put in your tea, I have every confidence in you.”

It wasn’t long before Ringo announced that the hidden cards were the 8, 4, 9, 10 and 2, in that order; once again, he was absolutely correct. John and Paul were flabbergasted, but no matter how much they begged George or Ringo, neither would utter a word of explanation.

“Ain’t he sweet?” said John to the others. “Ringo, how do you do it? I want you to tell me why you did five that time, and not four.”

“I’ve got a feeling—a feeling deep inside,” said Paul. “We can work it out... what you’re doing.”

“It won’t be long,” John agreed.

“Think for yourself,” was all George would say, over and over. The cards were put away, and Ringo sat magisterially behind his drum kit while the others tuned their guitars, so that they could have another go at Paul’s ‘Air Dish’ number, which they were having a lot of trouble agreeing on.

Two days later, after an especially fruitful studio session, the lads celebrated by going out for a late night meal at an Indian restaurant called *The Inner Light* that George recommended. As soon as the plates were cleared away, John produced a pack of cards.

“Have you heard the word?” he began, with that telltale twinkle in his eye. “This boy”—indicating Paul—“has been doing a lot of yogurt lately, and as a result his mental powers are razor-sharp right now. You should have seen him last night: he was really flying.” George looked sceptical.

“All I’ve got to do is get Ringo to help me,” John said, handing the pack to the drummer. “Give me back any five cards—any five at all.” A suspicious Ringo fanned through the pack, and picked out five cards, which he gave John. George watched intently from across the table. “Now, I’m going to show Paul *four* of these cards, but hide this one,” said John.

He slid one of the cards over towards Ringo, who was about to pick it up when Paul admonished him. “Let it be,” he said, putting the card under Ringo’s packet of cigarettes. He held up the other four cards above his head for all to see. They were the Ace of Spades, Four of Hearts, Nine of Spades and Four of Diamonds.

“Help!” said John with a giggle, looking at Paul. “I need somebody—not just anybody. I do appreciate you being around; won’t you please help me? What is the card beside Ringo?” The atmosphere was thick as everybody watched Paul, who screwed up his face in concentration. “Don’t let me down,” said John nervously. “Please please me, and get it right.”

At last, Paul spoke: “Is it the Two of Spades?” Ringo pushed his matchbox and cigarettes aside, and turned the card over. “Yes it is, it’s true,” the other three concurred in perfect harmony.

George and Ringo were baffled. How had Paul named that card? It could have been anything! “Do you want to know a secret?” asked Paul.

“Here’s another clue for you all, the adder is Paul,” offered John.

“One and one is two,” Paul teased, but the others were none the wiser.

A repeat performance was demanded, and Paul and John were happy to oblige. George chose five cards and handed them to John. As before, John looked at them carefully before putting one aside. He held the Queen of Clubs, King of Diamonds, Nine of Diamonds, and Five of Diamonds high above his head. *Floozy in the sky with diamonds*, he thought to himself.

“This one is harder,” said Paul, “But it’s on the tip of my tongue.” After a pause, he inquired, “The Five of Clubs?” He was, of course, correct.

“It’s all too much,” commented a thoroughly confused George.

“Ask me why I held up the cards the way I did,” said John. Ringo was looking at the sugar bowl longingly, so George took the bait. “Because”—John paused enigmatically—“Baby’s in black, and I’m feeling blue.” Everybody looked at the black queen. “Very helpful,” said George sarcastically, “As if I needed someone to tell me that!”

“That means a lot,” John insisted, “Would you rather I gave you no reply at all? I should have known better than to try to give you a hint.”

“Every little thing is important,” said Paul, trying to smooth the waves. “She’s a woman.” He pointed at the black Queen, and then at the other cards, adding, “Here, there and everywhere.” George stared ahead blankly.

“There’s a place, not far from here George,” Paul confided, “Where John and I sat down yesterday, just the two of us, and worked out that one together. It’s as good as the trick I remember you and Ringo doing for us the night before.”

“I’m off,” said a clearly exhausted Ringo, standing up and taking a pair of gloves off the table. “It’s been a hard day’s night.”

“Leave my mittens alone,” snapped John.

"Sorry," said Ringo, dropping them hastily. They walked outside.

"Misery," said John, buttoning up his coat. It was a dark and stormy night. "If the rain comes, run and hide your heads."

"Johnny, you can drive my car," offered Paul. "In spite of all the danger, George and I are going to walk."

"Though the taxman would probably like to tax my feet," George said, looking down. "These old brown shoes have seen better days."

"Run for your life," advised John, as the heavens opened.

"Now it's time to say good night," said Ringo. "Sleep tight. See you all tomorrow."

"Tomorrow never knows," said John cryptically.

"You won't see me," Paul said. "I'm meeting Dr. Robert, then I'm spending the afternoon with another girl. She says it's her birthday."

Weeks passed before the lads met again. They had been on separate holidays, but it was time to get back to work. They met for lunch on a dull grey day, eager to talk about some new material they had come up with.

"What goes on?" asked Ringo, as John and Paul came together. "I feel fine," said a particularly chipper-sounding John. "I'm down. I'm really down," moped Paul, letting his face grow long for a second before dissolving into laughter. "Would anybody like to see a card trick?" he asked a few minutes later, as soon as George had joined them. "Nobody I know," said John, who was keen to talk about several songs he had recently written. "Don't bother me," said George, who had songs of his own he wanted the others to consider. "Having been some days in preparation, a splendid time is guaranteed for Paul," joked Ringo, but nobody seemed to notice.

"Maybe we should do our new trick for no one," Paul said to Ringo, raising his eyebrows dramatically and riffing the cards noisily. "Roll up, step right this way; and that's an invitation. The magical mystery tour is waiting to take you away." John and George gave in and agreed to watch.

"With a little help from my friend," began Ringo, "I'd like to perform a new trick for you all. This one requires two people who are totally in tune to each others' every feeling, like a married couple. Paul, will you be my awfully wedded wife?"

"I will," replied Paul solemnly. He then grabbed Ringo's hand and added, "I want to hold your hand. Oh! Darling. Besame mucho."

"Honey, don't," Ringo said in mock consternation, backing off. "I wanna be your man." John and George laughed hysterically. "John, give Paul any four cards from this pack," said Ringo, as soon as he had regained his composure. Paul took the cards John offered him, glanced at them, and handed one back, saying, "From me to you." John grinned. Paul then placed the three remaining cards in a row on the table, but this time, only the middle one was face up.

"Tell me what you see," he asked Ringo. "The Ace of Hearts, in between two face-down cards," Ringo replied.

"What's this?" John asked, sliding the fourth card over next to the others. "The one after 909?"

After a short pause Ringo said, "I think it's the Three of Hearts."

"This happened once before," said John. "But you only had three cards to go by this time, not four like Paul and I did, and you can't even see two of them!" He threw the Three of Hearts on the table incredulously. "You can't do that," he said, shaking his head in disbelief, "Not every time. You really got a hold on me with this one."

"Within you, without you... is it a yin/yang thing?" asked George.

"I'd be an ass if I fell for that," John continued, ignoring him. "I detect a Spaniard in the forks." His incisive wit and fondness for spoonerisms were never far away. He turned over the two-face down cards—the King of Spades and the Eight of Clubs—and stared at them. *Everybody's got something to hide except me and my donkey*, he thought to himself.

"You've got to admit he's getting better," said Paul, who was clearly proud of Ringo's performance. "Getting better all the time."

"Couldn't get much worse," muttered John. "I'm just a jealous guy."

"Something in the way he grooves," George chipped in, "Attracts me like no other mother."

Of course they repeated the trick. George picked four cards and gave them to Paul, who handed one back to him, saying, "It's for you." Paul then placed the three remaining cards in a row on the table. This time, the first one was face down, followed by the Queen of Diamonds and the Ten of Hearts. "I've just seen a face," said Paul, pointing at the Queen. "Can't forget the time or place... she's just the girl for me." The others laughed.

"Oh, dear, what can I do?" asked Ringo, who seemed at a bit of a loss.

"Act naturally," advised Paul, tongue in cheek.

"Slow down," John suggested. "You're moving way too fast."

"What would you do if I guessed the wrong card? Would you stand up and walk out on me?" Ringo hoped he could carry that weight this time. In his nervousness, he edged closer to George and the hidden card.

"Get back," said Paul good-naturedly. Ringo retreated sheepishly.

"Come on. What card does George have?" asked John impatiently.

"I call your name," Ringo began, indicating the card with a flourish. "And you are—the Four of Spades!" He was relieved when George flipped the card over and proved him right yet again. "We hope you have enjoyed the show. All together now," said Paul, leading a brief burst of clapping.

"You're a legend in your own lunchtime!" beamed John.

It was starting to brighten up outside. "Here comes the sun; a cloudburst doesn't last all day," said George. "Good day, sunshine," Paul added with gusto. "Ah, Mr. Sun King," said John approvingly. "No more getting a tan standing in the English rain." They all felt good, in a special way.

"It took me so long to find out," Ringo mused. "But I found out!"

"When I get home I'll get it," said John. "I bet some of our earlier ideas come together in the end. I'll be back with the answer, I'll get you!"

"Remember the things we said today," said Paul. "Maybe I'm amazed at the way it fooled you, John. Another day, I'll explain it all to you."

"Love you to," George said. "Isn't it a pity it's so long, long, long?"

“It don’t come easy,” observed Ringo, looking at his hands. “After all that tricky card handling, I’ve got blisters on my fingers!”

“Imagine there’s no card tricks,” John shuddered. “No clubs, hearts, spades or diamonds; above us, only pi. Goo goo g’joob!”

“They’ve been going in and out of style, but they’re guaranteed to raise a smile,” Paul assured everybody.

“Across the universe, people have a real love of card tricks,” said John, who always wanted to have the last word.

“All you need is cards,” everybody chorused.

## Come and Get It

We now set off on the long and winding road which leads to full explanations of the above. Each trick involves communication between two people using only mathematical principles; there is no physical or verbal signalling. One person assumes the role of *performer*, choosing and displaying cards carefully, while the other is a *confederate*, surveying the scene later and doing some mental calculations, before correctly identifying a hidden card or cards.

**First trick:** Here is a verse to ponder.

In each list of ten there is bound,  
To be four that do rise; is that sound?  
In a paper with Erdős,  
By Gyorgy Szekeres,  
A counterexample is found.

This refers to an application—due to Erdős and Szekeres [?, page 124]—of the Pigeon Hole Principle.<sup>1</sup> Martin Gardner explained it in *Riddles of the Sphinx* in terms of “a row of 10 soldiers, no two of the same height. . . no matter what the order, there will always be at least four among the ten, not necessarily standing next to each other, who will be in ascending or in descending order” [?, page 5].

Hence, if we first fix an ordering on a set of ten distinct cards, and are given these cards mixed up, then there will either be four in ascending or four in descending order. To perform the trick, you and your confederate first decide on an ordering, e.g., you could use numerical ordering from 1 to 10. The cards are jumbled. If you see four in ascending order, deal all ten cards out from left to right and then turn over these four. Your confederate can tell which four cards are turned over by observing which six are visible, and since the order of the other four is known, each can be identified correctly. If there is no ascending subsequence of length four, then there is a descending one instead. The trick can be performed as above, simply dealing out the cards in reverse order; there are

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<sup>1</sup>If  $n$  pigeons fly over a piece of land which is broken up into  $k$  fields, where  $k < n$ , and each single pigeon does what pigeons do best, i.e., leaves a deposit, then at least one field will receive two or more deposits. The same holds for blackbirds or bluebirds. It is this principle which that cold water on the whole “Eight Days A Week” idea.

many ways to do this without arousing suspicion if you haven't shown anybody the cards in advance. Often one gets lucky, and finds runs of length five or six, that's what happened the second performance of this trick. Of course, the usual ordering from 1 to 10 is rather obvious. The ordering George and Ringo used was alphabetical—Ace, Eight, Five, Four, Nine, Seven, Six, Ten, Three and Two. (A version of this trick using “Erdős numbers” (but not the same old suit) may be found in Colm Mulcahy's online AMS article “Mathematical Card Tricks,” at <http://www.ams.org/new-in-math/cover/mulcahy1.html>.)

**Second trick:** The trick John and Paul performed for George and Ringo is often credited to mathematician William Fitch Cheney Jr. Martin Gardner has alluded to its 1950 appearance in a book by W. Wallace Lee [?] (see *Mathematics Magic and Mystery* [?, page 32] and *The Unexpected Hanging and Other Mathematical Diversions* [?, page 158]).

A volunteer selects five cards from a standard 52-card pack, and hands them to you so that nobody else can see them. You glance at them briefly, and hand one card back, which is set aside. You quickly display the remaining four card faces, in a row from left to right. Your confederate merely glances at the visible card faces, and promptly names the hidden card.

In each of John and Paul's performances, note that the first of the four cards displayed was the same suit as the hidden card. Indeed, the Pigeon Hole Principle guarantees that in any set of five cards there will be at least one suit match. If there are two Spades, let's agree to use the first position of the cards held up for the retained Spade—thereby revealing the suit of the hidden card. Since there are  $3! = 6$  ways to arrange the other three cards, we can communicate one of six things. The trick is to do this independently of the particular cards held. Even then, there is another issue to address if we hope to pull it off every time.

To get the permutations idea to work with any three cards, just note that they are distinct! Thus, with respect to a fixed ordering of the entire pack, one of them is *low*, one is *medium*, and one is *high*. This suggests an easy way to communicate a number between 1 and 6. For instance, try this CHaSeD ordering:  $A\clubsuit, \dots, K\clubsuit, A\heartsuit, \dots, K\heartsuit, A\spadesuit, \dots, K\spadesuit, A\diamondsuit, \dots, K\diamondsuit$ . Mentally label the three cards L (low), M (medium), and H (high) with respect to this ordering. Next, rank the six permutations of  $\{L, M, H\}$  as follows: 1 = LMH, 2 = LHM, 3 = MLH, 4 = MHL, 5 = HLM and 6 = HML. Now, order the cards from left to right according to this scheme to communicate the integer desired. Try this out to see out what integers John conveyed to Paul each time they did the trick. Are you certain that it happens all the time? After all, the hidden card could be any one of twelve Spades—try fixing a hole where the Ace gets in!

The last crucial observation we need is this: you must be careful as to exactly which Spade you hand back. Considering the 13 possible card values, 1 (Ace), 2, 3, ..., 10, J, Q, K, as being arranged clockwise on a circle, we can see that our two Spades are at most 6 values apart, i.e., counting clockwise one of them lies at most 6 vertices past the other. Give this “higher” valued Spade back to the victim, who hides it. You'll use the “lower” Spade and the other three

cards to communicate the identity of this hidden card. The first time the trick was done, the first card was the Ace of Spades, the integer communicated was 1, and as Paul pointed out to the others,  $1 + 1 = 2$ : The hidden card was the Two of Spades. The second time, the first card was the Queen of Spades (with a numerical value of 12), the communicated integer was 6, and  $12 + 6$  is equivalent to 5 (modulo 13). Sure enough, the hidden card was the Five of Spades.

One weakness in our method—especially if the trick is to be repeated—is the invariant use of the first position as the “suit-giver.” Here is one way to overcome this: since both you and your confederate get to see four cards, add their values and reduce modulo 4 (using 4 if you get 0), letting the answer determine the position of the suit-giving card. Thus, a Jack, 8, 2 and 7 would result in  $11 + 8 + 2 + 7 \equiv 0 \pmod{4}$ , so the fourth position would determine the suit-giver, and the other three cards communicate the permutation. Our method appears watertight, in that it does not seem to extend to packs with more than 52 cards. However, David and Hal Kierstead, Elwyn Berlekamp and others have noted that the trick generalizes to packs of size 124 cards [?]. Try it for an ordinary pack supplemented with one Lennonesque joker.<sup>2</sup>

**Third and Fourth tricks:** The last trick performed by Paul and Ringo is a kind of extension of the second one. Before attacking, we suggest relabelling this as the fourth trick, and first trying the following, simpler, “third” trick. You are given any five cards, one of which you hand back before placing the remaining four in a row on the table, *some face up, some face down*. Your confederate succeeds in identifying the fifth card. Hint: George made an insightful comment here, if only somebody would listen.

The Pigeon Hole Principle again guarantees that (at least) two of the five cards are of the same suit, let’s suppose it’s Spades. You hand one back, and use the remaining four to tell your confederate the identity of the fifth card. Use one particular position (e.g., the first) of the four available for the retained Spade—which determines the suit of the fifth card—and the other three positions for the remaining cards. The difference here is that you communicate using some kind of binary code—George *did* suggest a yin/yang principle—rather than permutations. Unlike in the earlier trick, the actual identities of any face up cards play no role! You can communicate any one of  $2^3 = 8$  integers in this way. Is this enough?

As before, save the “lower” Spade and communicate the identity of the “higher” one. Use a particular position (e.g., the first) for the retained Spade. Rather than indicating actual binary representations with the up/down arrangements, let’s agree on this convention: *UDD, DUD, DDU* (only 1st, 2nd or 3rd position is Up), and *DUU, UDU, UUD* (only 1st, 2nd or 3rd position is Down), respectively, reveal to your confederate which of 1, 2, 3 or 4, 5, 6 they should add to the lower Spade value.

Finally, consider the last trick that Paul and Ringo did for the others. Note that there may not be any suit matches among four cards! Start by repartitioning the pack into three new Suits of 17 cards, leaving one card (say  $A \diamond$ )

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<sup>2</sup>Don’t you think the joker laughs at you?

aside. The new Suits are the standard suits ♣, ♥, ♠, each supplemented with four ♦'s: Suit A is A♣, 2♣, . . . , K♣, 2♦, 3♦, 4♦, 5♦; Suit B is A♥, 2♥, . . . , K♥, 6♠, 7♦, 8♦, 9♦; and Suit C is A♠, 2♠, . . . , K♠, 10♦, J♦, Q♦, K♦. If one of the four cards is A♦, play the others face down, and watch the audience reaction as your confederate demonstrates some *real* magic powers! Otherwise (at least) two of the cards are from the same new Suit. Let's assume it's Suit A.

Retain the “lower” card and hide the “higher” one, whose numerical value is  $k$  past the retained card, where  $k$  is between 1 and 8 this time. As before, the displays  $UDD$ ,  $DUD$ ,  $DDU$ ,  $DUU$ ,  $UDU$ ,  $UUD$  can communicate  $k$  if it's between 1 and 6. Since at least one card will be face up, we can use such a card—or the first such if there are two—to reveal the suit at the same time! However, we also need a way to communicate 7 or 8, and for this we use the  $UUU$  option. Let's agree that one particular  $U$  (say, the middle one) gives the suit. Then with respect to some ordering of the pack—such as lining up Suits A, B, C—there are two ways to play the other two: Low-High (to convey  $k = 7$ ) or High-Low (for  $k = 8$ ). You should now verify that Paul was indeed able to communicate the hidden card to Ringo in the last trick they did together.

Michael Trick at Carnegie Mellon kindly put together a website which illustrates this—er—*trick* in action, see: <http://mat.gsia.cmu.edu/CARD/>. It uses a slightly different suit convention: the three basic suits are Clubs, Diamonds and Hearts, each supplemented with four Spades.

This concludes our look at four fab four four-card tricks.<sup>3</sup>

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<sup>3</sup>to the tune of “When I'm Sixty-Four”