

IBM Ring 9 Teach-a-Trick Session
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Mathematical & Self-Working Card Tricks

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1 Binary 101

Effect: The victim calls out a random number, say between ten and thirty. Count out that many cards from the deck and hand them to the victim, who then does the “Down Under” deal until one card remains. You correctly name that card.

Method: You must know the top card on the deck at the outset. Then, you need to mentally compute a number used to do the count, which is actually done in two installments. Find twice the difference between the number called out and the highest power of two which is strictly less than it. For instance, if fourteen is called out, then compute $2 \times (14 - 8) = 12$. Now count out that many (twelve in this case) cards casually, dealing into a pile, and thus reversing them. Gather the cards while feigning poor memory, saying “How many did you say? Oh, fourteen,” and scoop the extra two cards off the deck and place them *beneath* the cards in your hand. Thus you now have the card whose identity you know in position twelve in a packet of fourteen cards. If a power of two, e.g., sixteen, is called out, then since $2 \times (16 - 8) = 16$, simply count out all sixteen cards into a pile, reversing them. In either case, hand this packet to the victim, and carefully direct the Down Under dealing. The last card is guaranteed to be the original top card.

Mathematics: Everything becomes clearer (or not...) if we convert to base 2. First suppose the called out number is $1abc\dots e$ in base 2, and is not itself a power of 2. The highest power of 2 less than this is $1000\dots 0$, which upon subtraction from $1abc\dots e$ yields $abc\dots e$. Twice that is $abc\dots e0$. You count out that number into a pile, thus reversing their order, and finally scoop the remaining cards underneath. This buries the original top card $abc\dots e0$ deep in a pile of size $1abc\dots e$. If a power of two, say $1000\dots 0$, is called out, then the dealing out puts the original top card at the bottom of a pile of this size. The reader should now check that in both cases, the Down Under deal results in the last card remaining in the hand being the known one.

Source: Gardner mentions learning the basic idea behind this one from Mel Stover in *The Unexpected Hanging and Other Mathematical Diversions* [?, page 152 and 155-6], and credits John Scarne with the above presentation. He also mentions a Bob Hummer version from 1939, and describes several variations on the theme due to Sam Schwartz and others.

2 Sensitive Fingers

Effect: The victim divides the deck into two piles, and hands one to you behind your back or under the table. You claim that you are learning how the cards feel when you cannot see them, then bring them forth, turn them over publically, and have these face-up cards shuffled

into the other face-down pile. You hide the cards from view again, and then bring forth a pile that you hand to the victim, commenting that even though you obviously cannot see which cards are face-up, your acute sense of touch rarely lets you down. You then predict that you've given the victim just the right cards so that each of you now has the same number of face-up cards. Sure enough, when you bring your cards forward, and the two piles are checked, your prediction is seen to be correct.

Method: After the deck is split for the first time, and one pile is in your control but hidden from view, simply count how many cards you have. Now bring them forward, flip them over, and have them shuffled into the victim's cards as above. When you get the deck back, secretly count off the same number of cards as you had earlier, flip them all over as one unit, bring them into view and hand them to the victim as you make your prediction. The counting of face-up cards in each pile will confirm the accuracy of your prediction.

Mathematics: Suppose the pile you are handed has s cards in it. These are flipped over and shuffled into the remaining cards. When you get the deck back, you count off s cards, flip them, and hand them to the victim, retaining the rest. If the victim now has t face-up cards, for some $t \leq s$, then they have $s - t$ face-down cards, but these were face-up until you flipped their pile over! This leaves you with the other $s - (s - t) = t$ face-up cards. The beauty of this is that it works without any knowledge of the value of t .

Source: This is very similar to a trick in Martin Gardner [?, page 97], with the handling here changed to conceal the fact that the deck is split in the same ratio twice.

The special case where the deck (or a subset thereof) is carefully (and usually publically) halved on each occasion, is due to Bob Hummer, and is well known [?, page 165]. Bob Longe has a version called "Behind My Back" in [?, page 53]. Of course, in that case it does not matter which pile is turned over at the end, and both piles will then have the same number of face-down cards. However, while this one stumps a general audience, the more alert victim will see through it pretty quickly if the whole deck is used: they'll be very surprised if the trick starts with 26 face-up cards out of 52 and you both end up with only 11 face-up cards!

This trick is closely related to a discrete version of the "Water and Wine" or "Coffee and Cream" puzzle that appeared in early Martin Gardner collections [?, Chapter 6], [?, Chapter 13], which we state here in the following form: Start with a cup of oil and a cup of water, remove a spoonful of oil from the former and dump into the latter, then transfer a spoonful of liquid from the second container back to the first one, so that both containers again contain exactly one cup of liquid (we ignore chemical interactions!). A question naturally arises: does the water cup contain more oil than the oil cup contains water? Note that it was never specified if any stirring took place! The percentage of "contaminant" in each container is unknown, but the same: if the first container is now $x\%$ oil, then it is $(100 - x)\%$ water, and so the other must be $x\%$ water and $(100 - x)\%$ oil.

As Martin Gardner points out [?, page 97], mathematics can easily get in the way here, obscuring the fact that the only thing required in order to draw a similar conclusion is, that at the finish, each container have the same amount of liquid in it as it had at the start. The containers need not be the same size, and the nature and frequency of the liquid transfers is irrelevant. Forget percentages: if v cubic units of the liquid which ends up in the second container was originally in the first, clearly it's replacing v cubic units of liquid which started in the second and is now in the first.

The card trick uses the same principle, with the flipping over of a pile at the end effectively changing water into wine (and vice versa).

3 Pick Nine Cards, Any Nine Cards

Effect: Have the victim select any nine cards from the deck, setting the rest aside. This packet is shuffled freely while you turn away. Ask the victim to look at and show around the third down from the top, then replace it. Now you direct the silent spelling out of the card's name, one card of the nine being moved for each letter. Finally you turn back, take the cards, place them behind your back and after some concentration, successfully bring forward the chosen card.

Method & Mathematics: Each card name had three words, ranging in length from ace of clubs to three of diamonds. The spelling proceeds for each of these words separately as follows: cards are dealt from the top onto a pile on the table, one card for each letter in the individual words, and *at the conclusion of each word*, these cards are picked up as a unit and placed underneath the remaining cards. (Each of these spellings reverses the order of some of the cards, this is what makes the trick work.) For instance, if the chosen card is 7♣, the victim spells out S-E-V-E-N while dealing five cards into a pile, picks them up and puts them under the remaining cards, then spells O-F, dealing out two cards, puts those under the remaining cards, and finally spells C-L-U-B-S, dealing out five cards again before putting them under the remaining cards. (It is a very good idea to demonstrate this spelling technique first, before you turn away, *using ten or more* cards borrowed from the rest of the deck, and using a different card, perhaps the second one, as the chosen one. Victims are apt to get the dealing/spelling/replacing directions wrong given half a chance. Don't use nine cards in your run through, in fact, make it seem like the number of cards is irrelevant.)

Now turn around again and take the cards behind your back, stressing that you had no control over which cards were used, or which of those was chosen. Remind the audience that the names of the cards vary in length a great deal, from short ones like 10♣ (ten letters in total) to long ones like Q♦ (fifteen letters). With the cards behind your back, all you have to do is locate the middle one, which is fifth from either end of the packet. Bring this forward, and ask what the chosen card was right before you turn it over.

It's a fun and elementary exercise to check why this works. Stressing the variable length of the card names seems reasonable, but it's pure misdirection: the individual word spellings/card reversals result in the chosen card being in the fifth position after the first two of the three words have been spelled, and the third spelling does not disturb it!

For a repeat performance, you can suggest that the victim deliberately lie about the name of the card while spelling. This strategy is a two edged-sword: while it tends to baffle a lay audience, it alerts a mathematically inclined one to the invariance inherent in the trick.

It's easy enough to conceal the fact that the chosen card must start in the third position: have the victim chose the top card instead, after shuffling, then put the cards behind your back under some pretext, and transfer two cards from the bottom to the top before the dealing begins. For instance, you can pretend to be getting a tactile impression from the chosen card, closing your eyes and squinting dramatically at this point. You can repeat these theatrics during the concluding production of the card from behind your back.

Source: This wonderful Jim Steinmeyer invention is in a recent Gardner volume for children as “Nine-Card Spell” [?, page 76]; and, in a more involved form, in Longe [?, page 40] as “The Impossible Nine.” In the latter, the victim peeks at the bottom card of the nine, and then the Under Down deal is used to move it to the third position.

4 Twentieth Century Mind Reading

Effect: The victim hides an unknown number of cards taken from a full deck, and uses that number to determine (but not remove) one card from the remainder. Taking the deck back, you claim that you’ll be able to *feel* the selected card. After running more than half of the deck face down from hand to hand, you admit defeat and resort to mind reading. It works!

Method: Invite somebody to shuffle the deck, and have the victim decide on an integer n between 1 and 20, discreetly remove that many cards, and pass them around so that others can secretly count them. This is the key number, which everybody but you knows, and the removed cards remain hidden from you for the remainder of the performance. Now take back the rest of the deck, and show the audience the faces of the first 20 cards, one by one, while retaining their order, instructing one and all to spot and remember the n^{th} card, but not to react in any way when they see it go by. Put those 20 cards down, and ask for a random number, perhaps saying “about half of what I have left here” holding up the balance of the cards. If somebody calls out “Seven,” count out 7 cards, place the packet of 20 on top of them, and drop the lot on top of the remaining cards, remarking “It’s important that I don’t know how many cards are left, because then I could work out how many you hid at the start, and we don’t want that do we?!”

Now comes the show business: start passing cards from the top of the deck, in your left hand, to your right hand, again retaining the order. Make it clear that you cannot see the faces as you pretend to “feel” them, but don’t let anybody else see them either. What you *really* do is silently count to 52 as you pass cards, starting with $20 + 7 + 1 = 28$ (“28, 29, . . . , 52”). The 52^{nd} card passed *is* the chosen card, and it is now on the bottom of the pile in your right hand! Place the cards in your left hand on top of those in your right hand, thus bringing the chosen card to the bottom of the deck, while distracting the audience by remarking “It’s not working, I just don’t have a feel for your card today. Besides, you sure don’t want to let me get to the end of the deck in case I was trying to figure out your secret number!” As you say this, square up the deck by tapping it on the table, ensuring that you and you alone get a peek at the bottom card. Cut the deck, riffle-shuffle, and push it to one side. Suppose the chosen card you just spotted is $8\clubsuit$, proceed something like this: “Let’s resort to mind reading . . . the lady in the back with the glasses is giving me vibes . . . a black card you say?” Even though you had a 50% chance of getting that right, the audience is likely to grow quiet and hang on your every word at this point. Look at somebody who looks like they are muttering something skeptical to a neighbor and point, saying “Hmm, you’re telling me it’s a high-valued black card—and even!” Look at somebody else, “You sir, are giving me definite ‘eight’ vibes. So it’s a black eight!” Complete the revelation quickly with the “help” of somebody else.

Mathematics: Suppose that n cards are hidden at the outset. Then the chosen card is at position n in the packet of 20 counted off the deck of $52 - n$ cards. The remaining

$52 - n - 20 = 32 - n$ cards are then split into two packets, one of size m (in response to the audiences suggestion), the other necessarily of size $32 - n - m$. The packet of 20 is placed on top of the packet of m , and the remaining $32 - n - m$ are dropped on top of these. This puts the chosen card at position $(32 - n - m) + n = 32 - m$ from the top, and since you know m , all is well! Noting that $52 - (20 + m) = 32 - m$, you can simply start counting at $(20 + m) + 1$ until you reach 52 to reach the chosen card.

Source: This is adapted from a trick of the same name in Patton [?, page 45], where the performer never touches the cards at all. Other endings suggest themselves, apart from the bogus “feeling” or mind reading. You could dream up a long phrase to spell out to get to the chosen card, if you can do that kind of thing on the fly, or bring the card to the top of the deck and then keep it there through a few riffle shuffles and finally produce it from behind your back.

Note that in our version, nobody has any idea that you ever peeked at a card, in fact as people try to analyze the trick afterwards, they frequently swear that you didn’t. Also, few realize that it is possible to locate their card without knowing the secret number (BTW, you *still* don’t know it!). Ask for the hidden cards back before you forget, and count them before somebody says “You never told us *how many cards* were hidden!”

References

- [1] Martin Gardner, *Hexaflexagons and Other Mathematical Diversions*, University of Chicago Press, 1988, originally published as *The Scientific American Book of Mathematical Puzzles and Diversions*, Simon and Schuster, 1959
- [2] Martin Gardner, *The 2nd Scientific American Book of Mathematical Puzzles & Diversions*, Simon and Schuster, 1961
- [3] Martin Gardner, *The Unexpected Hanging and Other Mathematical Diversions*, University of Chicago Press, 1991, originally published by Simon and Schuster, 1969
- [4] Martin Gardner, *Mental Magic*, Sterling, 1999
- [5] Bob Longe, *World’s Best Card Tricks*, Sterling, 1991
STARTS OFF WITH A CRASH COURSE IN SLEIGHT OF HAND, BUT HAS MANY SOLID MATH-BASED TRICKS AS WELL.
- [6] Bob Longe, *World’s Greatest Card Tricks*, Sterling, 1996
ANOTHER ONE WHICH STARTS OFF WITH SLEIGHT OF HAND HINTS, BEFORE MOVING ON TO INCLUDE MANY MATH-BASED TRICKS.
- [7] Temple Patton, *Card Tricks Anyone Can Do*, Castle Books, 1968
SUBTITLED *A Mathematical Approach to Card Magic* ON THE COVER PAGE, THIS LONG OUT-OF-PRINT BOOK WAS IN THE SAME SERIES AS SUCH SEMINAL WORKS AS *How To Get Along With Yourself*, *Let’s Decorate Your Home*, AND *The Meaning Of Your Dreams*.
- [8] William Simon, *Mathematical Magic*, Dover, 1993, originally published by Charles Scribner’s & Sons, 1964
THE CURRENT EDITION HAS A 1993 INTRODUCTION BY MARTIN GARDNER, WHO NOTES THAT THIS WAS THE SECOND “NONTECHNICAL COLLECTION OF EASY-TO-DO, SELF-WORKING TRICKS BASED ON MATHEMATICAL PRINCIPLES” (HIS OWN *Mathematics Magic & Mystery* HAVING BEEN THE FIRST). THE LAST CHAPTER HAS SOME FINE CARD TRICKS.